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Simulation of Josephson antenna in 3D space

V.V. Kurin, A.M. Klushin, I.A. Shereshevskii, **N.K. Vdovicheva**
Institute for Physics of Microstructure RAS, Nizhnii Novgorod, Russia;
Nizhnii Novgorod State University, Nizhnii Novgorod, Russia;
e-mail: hope@ipmras.ru

We present the mathematical model and algorithm for simulation of active Josephson antenna, which consists of a few lumped Josephson junctions and sources of bias voltage, connected by perfectly conducting wires placed on dielectric substrate. For simple model of such antenna, we present some results of simulation, in particular, current-voltage characteristics of junctions and examples of antenna patterns.

1 Introduction

The active Josephson antenna is a system of half-wave vibrators, power supply circuits including sources of d.c. voltage or current, Josephson junctions as active elements producing high-frequency signal. The system can lie on dielectric substrate. All components of the system interact each other through the common power supply circuit as well as electromagnetic emission. Since the Josephson junction is highly non-linear element, interaction has a strong impact on the temporal and spatial emission spectra, in particular carrier frequency and antenna pattern. Due to non-linearity, this interaction can not be described properly in terms of transfer functions even in the simplest case of single-element antennas, which necessitates the use of numerical simulations to study such systems. The existing packages for simulation of electromagnetic systems such as CST-Studio [1] do not allow simulation of such system because they do not support all necessary types of nonlinear elements, in particular, lumped Josephson junctions. Note also that the formulation of simulation problem itself seems to be nontrivial.

We use for simulation the discrete model of Maxwell equations known as Yee scheme [2, 3]. To simulate the temporal dynamics of electromagnetic field, we use explicit Euler method. The combination of Yee scheme and Euler method is known as Finite Difference in Time Domain Method (FDTD) [2]. The Perfectly Matched Layer (PML) boundary conditions [2, 4] are used to avoid the reflection of electromagnetic waves on the boundaries of the calculating region. We take into account the perfectly conducting wires of power supply circuit assuming the corresponding components of electric field are zero on the edges of Yee grid belonging to these wires. The Josephson junctions are simulated by the short segments of wires on which the electric field, curl of magnetic field and superconducting phase are connected by the system of equation similar to Josephson equation for lumped Josephson junction [5]. The sources of d.c. voltage are described in a similar way. By using Fourier analysis of Josephson current versus time, we determine the optimal frequency of electromagnetic antenna emission. To calculate the antenna pattern for this frequency, we use 3D near-to-far-field transformation [2] of the Fourier harmonics of electric and magnetic fields on the surface surrounding the antenna.

2 The Yee scheme, PML boundary conditions and lumped elements

In this section, we briefly discuss the basic features of the model of active Josephson antenna which we use for simulation.

2.1 The 3D Yee scheme

The Yee scheme [2, 3] is the discrete model of Maxwell equations for electromagnetic field. In this model, the rectangular grid (possibly, nonuniform) in 3D space is considered. As it is shown in Fig. 1, the components of electric field are the functions on the corresponding *edges* of the grid and the components of the magnetic field are the functions on the grid *faces*. The grid analogs of the Maxwell equations are of the form

$$\begin{aligned} \varepsilon\frac{\partial E_x(i,j,k)}{\partial t} &= (\widehat{\mathbf{rot}}\,H)_x(i,j,k) - \frac{1}{S}J_x(i,j,k), \\ \mu\frac{\partial H_x(i,j,k)}{\partial t} &= -(\widehat{\mathbf{rot}}\,E)_x(i,j,k). \end{aligned} \tag{1}$$

Here (i,j,k) are the numbers of grid edge, ε, μ are the permittivity and permeability of the medium, J_x is component of electric current, S is the square

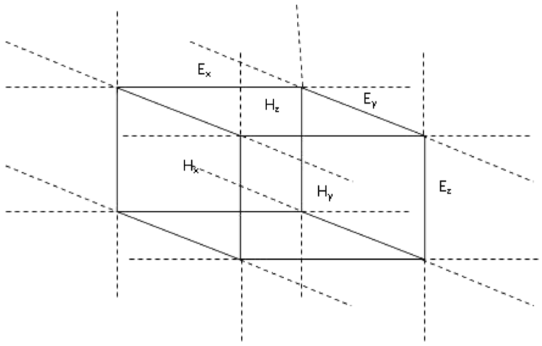

Figure 1: The Yee grid.

of grid face. Operators in the right parts of the equations are defined as follows:

$$(\widehat{\mathbf{rot}}\,H)_x(i,j,k) = \frac{H_z(i,j,k) - H_z(i,j-1,k)}{dy_H} - \frac{H_y(i,j,k) - H_y(i,j,k-1)}{dz_H},$$

$$(\widehat{\mathbf{rot}}\,E)_x(i,j,k) = \frac{E_z(i,j+1,k) - E_z(i,j,k)}{dy_E} - \frac{E_y(i,j,k) - E_y(i,j,k+1)}{dz_E}.$$

The equations for other field components look similar. These equations are solved by the explicit Euler method in two steps. First, we calculate the values of electric field at time $t + \delta t$ using in the right part of the equation the values of magnetic field at time t. Then, the obtained values of electric field are used to calculate magnetic field at the next time moment. It can be shown that, for sufficiently small time step, this procedure is stable. The described combination of the Yee scheme and Euler method is known as FDTD method [2].

2.2 On the PML boundary condition in 3D

Another problem appearing in the simulation of the most electrodynamic systems is connected with unboundedness of the region in which electrodynamic waves are propagated. To overcome this difficulty, one must consider some *bounded* region such that there are no reflections of electromagnetic waves at its boundaries. To this goal, J. Beringer [4] suggests to surround a calculating area with a layer of some *artificial* medium, in which the electromagnetic waves satisfy the so-called *splitted* Maxwell equations.

It was shown in [4] that the reflection of plane electromagnetic waves is absent on the flat boundary between usual and Beringer's media for all frequencies and wavenumbers, and the fields versus the distance from the boundary decrease exponentially. So, the flat layer of such artificial medium was called *Perfectly Matched Layer* or PML. We do not discuss here the details of numerical implementation of this method because they are well described in [2] and, for 2D case, in our previous work [5].

2.3 The lumped elements: Josephson junctions and bias voltage sources

Now we discuss how to take into consideration the lamped elements of antenna, namely, Josephson junctions and bias voltage sources. Recall that such elements of antenna as vibrators and power supply circuit in our model consist of perfectly conducting wires. To simulate these elements, we suppose that some grid edges belong to the wires and for these edges we set to zero corresponding components of electric field. It follows then from the equation (1) that the electric current through such edges is equal to the corresponding component of magnetic field curl.

We simulate the bias voltage source also as the edge of grid such that electric current through this grid edge is given by the relation

$$J_s = -\frac{\mathcal{E}_s - E(\mathbf{x}^s)d_s}{r_s}\ , \tag{2}$$

where $\mathcal{E}_s$ is the electromotive force of source, r_s is its inner resistance, $E(\mathbf{x}^s)$ is the component of electric field on the edge and d_s is the edge length. The evolution of electric field component $E(\mathbf{x}^s)$ is then described by the equation (1) in which current J is given by the relation (2).

The current from grid edge containing Josephson junction is given by Josephson relation [6]

$$J_j = C_j \frac{dE(\mathbf{x}^j)d_j}{dt} + \frac{E(\mathbf{x}^j)d_j}{R_j} + I_{c,j}\sin\varphi_j,$$

where R_j, C_j are the resistance and capacity of junction respectively, d_j is the length of grid edge, $I_{c,j}$ is critical current of junction and φ_j is superconducting phase. In its turn, the superconducting phase satisfies the equation

$$\frac{d\varphi_j}{dt} = \frac{2\pi d_j}{\Phi_0} E(\mathbf{x}^j),$$

where Φ_0 is quantum flux [6]. By substituting the expression for Josephson current into the equation

(1) we obtain the connected system of equations for electric field and phase on the edge. This system is solved by the semi-implicit method similar to ones described in [5].

3 Determination of the emission frequency and calculation the antenna pattern.

Due to the non-linearity of the Josephson junctions, the system in consideration emits the wide spectrum of frequencies. To find the frequency that is emitted most efficiency, we use the spectrum analysis of the current versus time through one of the junctions. The frequency at which this spectrum achieves the maximum value, we refer as *emission frequency* and use it to calculate the antenna pattern.

Recall that the antenna pattern is defined by the values electromagnetic field on the infinity sphere, while the FDTD method defines the fields only in the bounded region in the antenna neighborhood. To solve this problem, the so-called *Near-To-Far-Field transformation* is usually used [2].

To describe this transformation, let us denote by $\xi = x/|x|$ the unit vector from the origin to the infinity observation point. Let further

$$\tilde{E}(\xi) \equiv \lim_{|x|\to\infty} |x| e^{-ik_0|x|} E(x),$$
$$\tilde{H}(\xi) \equiv \lim_{|x|\to\infty} |x| e^{-ik_0|x|} H(x),$$

where $k_0 = 2\pi f_0/c$ is the wavenumber corresponding to the emission frequency f_0 and $E(x)$, $H(x)$ are the complex amplitudes of the electromagnetic field at the point x for the frequency f_0. The antenna pattern $D(\xi)$ is the absolute value of the *Poynting vector*, which is defined by the values $\tilde{E}(\xi)$, $\tilde{H}(\xi)$ as

$$P(\xi) = \frac{1}{2}\mathrm{Re}\big[\tilde{\mathrm{E}}(\xi), \tilde{\mathrm{H}}^*(\xi)\big].$$

It can be shown by using the vector Kirchhoff integral [7] that the following expressions

$$E(\xi) = \frac{ik_0}{4\pi}\int_{\partial\Omega} e^{-ik_0(\xi,x_0)}(-[[n,E],\xi] + [n,H]),$$
$$H(\xi) = -\frac{ik_0}{4\pi}\int_{\partial\Omega} e^{-ik_0(\xi,x_0)}([[n,H],\xi] + [n,E])$$

hold. Here $\partial\Omega$ is the boundary of the region Ω which contains all inhomogeneities of our system, n is internal normal to $\partial\Omega$, and E, H are the complex amplitudes of the fields at the emission frequency.

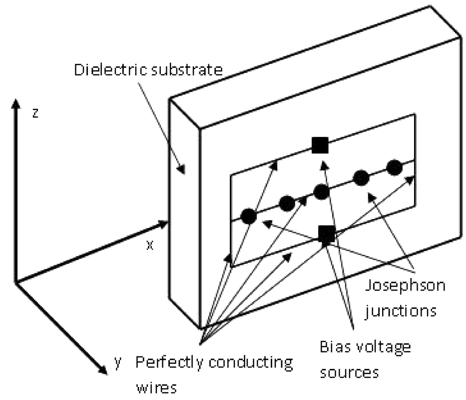

Figure 2: The scheme of antenna.

In the numerical implementation, the region Ω is the cuboid made up of cells of the Yee grid such that it contains the antenna and its boundary consists of grid faces. To calculate the tangential component of the complex amplitudes of fields on these faces, we use the linear interpolation over the neighbor edges or faces for electric and magnetic field correspondingly. To approximate the integrals, we use the rectangle method.

4 Some numerical results

In this section, we present some results of the simulation of the active Josephson antenna showed schematically in Fig. 2. This antenna consists of five Josephson junctions and two bias voltage sources to avoid asymmetry of the emitting system with respect to z-axis. We consider two cases of the system: (i) the permittivity of dielectric substrate is the same as for free space (vacuum); (ii) the permittivity is equal to 10 which corresponds to silicon substrate. In Fig. 3 the current-voltage characteristic of first junction for the case (i) is shown. The one for another junctions as well as for case (ii) are not significantly different. In Figs. 4, 5 the emitted power versus bias voltage for the cases (i) and (ii) correspondingly are shown. One can see the significant effect of the substrate permittivity on the characteristic of antenna. We note in this regard that dielectric also affects the optimal emission frequency unlike the current-voltage characteristic.

Finally, in Figs. 6, 7 the examples of antenna patterns for the same bias voltage for two cases are shown. As it was mentioned above, the optimal emission frequencies are slightly different, but the antenna patterns differ dramatically.

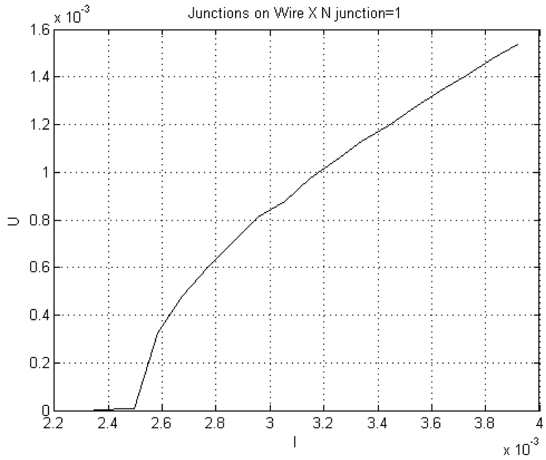

Figure 3: The current-voltage characteristic of first junction.

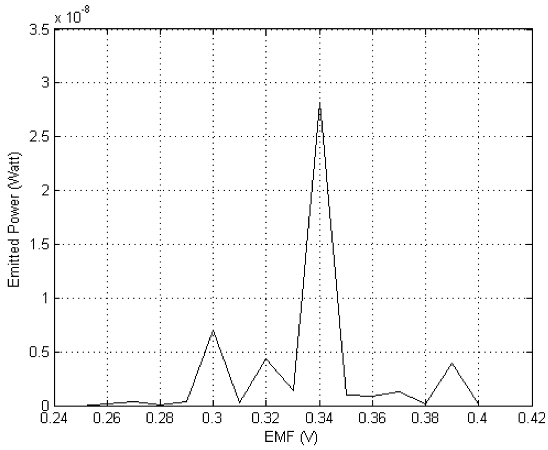

Figure 4: The emitted power versus bias voltage, substrate permittivity 1.

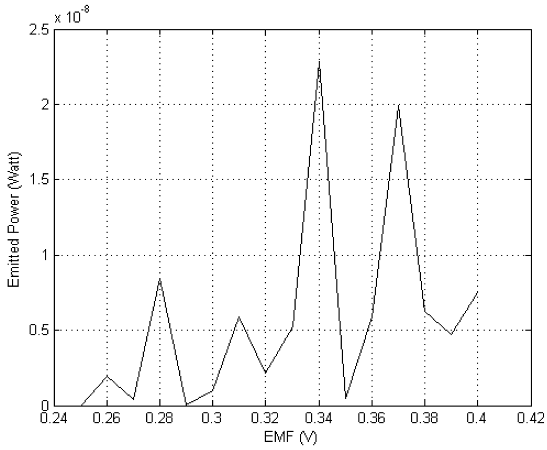

Figure 5: The emitted power versus bias voltage, substrate permittivity 10.

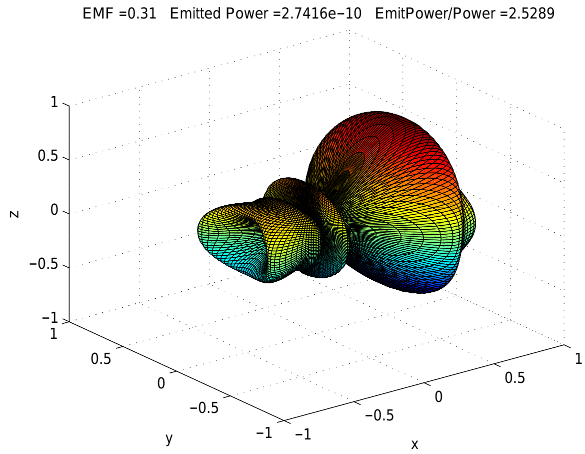

Figure 6: The antenna pattern at the bias voltage 0.31 V, substrate permittivity 1. The emission frequency 437 GHz.

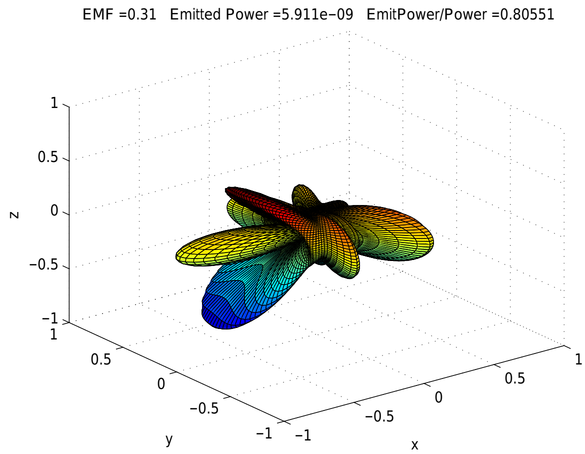

Figure 7: The antenna pattern at the bias voltage 0.31 V, substrate permittivity 10. Emission frequency 437 GHz.

5 Conclusion

We develop the numerical method and algorithm to simulate three-dimensional models of the Josephson antennas. The algorithm is based on the three-dimensional Yee scheme for the Maxwell equation and FDTD method for the corresponding difference equations solution. The software implementation allows us to create antenna models, to calculate the current-voltage characteristics of the Josephson junctions in antenna, and to determine the optimal emission frequency, the antenna pattern and the emitted power.

We found in numerical experiments that for simple model of antenna its characteristics depend essentially on the substrate permittivity. Besides, we found that emitted power depends in complex way on the bias voltage.

Acknowledgments

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