

Spin current and second harmonic generation in non-collinear magnetic systems: the hydrodynamic model

E A Karashtin^{1,2}  and A A Fraerman^{1,2}

¹ Institute for Physics of Microstructures RAS, GSP-105, 603950, Nizhny Novgorod, Russia

² University of Nizhny Novgorod, 23 Prospekt Gagarina, 603950, Nizhny Novgorod, Russia

E-mail: eugenk@ipmras.ru

Received 28 November 2017, revised 4 March 2018

Accepted for publication 9 March 2018

Published 26 March 2018



Abstract

We report a theoretical study of the second harmonic generation in a noncollinearly magnetized conductive medium with equilibrium spin current. The hydrodynamic model is used to unravel the mechanism of a novel effect of the double frequency signal generation that is attributed to the spin current. According to our calculations, this second harmonic response appears due to the ‘non-adiabatic’ spin polarization of the conduction electrons induced by the oscillations in the non-uniform magnetization forced by the electric field of the electromagnetic wave. Together with the linear velocity response this leads to the generation of the double frequency spin current. This spin current is converted to the electric current via the inverse spin Hall effect, and the double-frequency electric current emits the second harmonic radiation. Possible experiment for detection of the new second harmonic effect is proposed.

Keywords: second harmonic generation, spin current, non-adiabatic spin transfer, spin Hall effect, spin–orbit coupling, hydrodynamic model, non-collinear ferromagnet

(Some figures may appear in colour only in the online journal)

1. Introduction

Spin current is the flow of spin. It is often accompanied by the flow of charge, i.e. the electric current [1]. Systems with such current are widely studied due to the possibility to electrically switch the magnetic state [2–5]. This mechanism is commonly used when the spin injection is needed [6–8]. Another way to produce the spin current is spin pumping [9, 10]. Here, the spin current is created with no average charge motion. Such spin flow is called the *pure* spin current [11]. An important effect that appears due to the pure spin current is the inverse spin Hall effect [12–15]. It consists in the appearance of the charge current which is perpendicular to the externally created spin current. The mechanism of the inverse spin Hall effect is the spin–orbit interaction [12]. It provides the possibility to detect the pure spin current. Other possibilities include the x-ray magnetic circular dichroism measurements

[16], the electron spin resonance method [17], or the methods that utilize the spin-valve configuration [7, 18–20].

A special kind of systems with the spin current is a system where the pure spin current exists in equilibrium [11, 21–26]. One may show that if the magnetic system is considered the magnetization distribution should be non-collinear in order to have an equilibrium spin current [21, 27–30]. Such systems attract attention due to new interesting properties that may be caused by the spin current. Indeed, both the inversion symmetry and the time reversal symmetry are broken in equilibrium here. This leads to several new phenomena. The flexo-magnetoelectric effect was predicted in non-collinearly magnetized media [31–36]. It will hopefully provide a new way to control the magnetization without applying a magnetic field. Another predicted effect is the second-order nonlinear optical effect due to the spin current [37]. It was observed in the GaAs semiconductor where the spin current was induced

by simultaneous illumination with two laser pulses [38]. Such effect may be used as a powerful tool for spin current diagnostics. This paper is devoted to the investigation of the described second harmonic generation effect in a non-uniformly magnetized medium with equilibrium pure spin current. The microscopic reasons of this effect are discussed in the framework of the simple hydrodynamic model describing the conduction electrons. A possible experiment for detection of the predicted effect is proposed based on the polarization properties of the second harmonic signal.

2. Symmetry considerations

We consider a magnetic medium with the coordinate-dependent magnetization $\mathbf{M}(\mathbf{r})$ normalized to unity ($|\mathbf{M}| = 1$). We find the equilibrium spin current J_{jk} , j being the spin index and k the coordinate one. Such current should change its sign under the space-inversion operation [11]. Besides, the exchange interaction is strong in ferromagnets and thus we are seeking for the exchange spin current. The symmetry of the exchange energy requires that the spin current transform under the rotation of the whole spin system so that its spin component is rotated. In this approximation, the tensor J_{jk} may be written as [33, 39]:

$$J_{jk} = u \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_k} \right]_j, \quad (1)$$

where u is a constant; j is the spin index of the spin current tensor, k is its coordinate index. Note that we suppose that the magnetization changes slowly in space ($\left| \frac{\partial \mathbf{M}}{\partial x_k} \right| \ll \frac{1}{a}$, a being the lattice constant in the medium, x_k the k th coordinate in the Cartesian coordinate system). So we take into account only the first-order derivative of \mathbf{M} and neglect the higher derivatives.

As it was mentioned above, the spin current tensor changes its sign when the coordinate inversion operation is applied to the system. It may exist only in non-centrosymmetric media. The spin current determined by (1) exists in a special kind of media in which the inversion symmetry is broken due to the non-centrosymmetric spatial distribution of magnetization $\mathbf{M}(\mathbf{r})$. This tensor (1) may therefore be used to describe the non-centrosymmetric properties of the magnetic moment distribution.

The second harmonic generation is also possible only in non-centrosymmetric media [40]. The polarization at double frequency is usually written as

$$P_i^{2\omega} = \chi_{ilp} E_l^\omega E_p^\omega, \quad (2)$$

here E_l^ω (E_p^ω) denotes the l th (p th) component of the electric field \mathbf{E}^ω of the wave propagating in the medium, χ_{ilp} is the double frequency permittivity tensor. The left-hand part of (2) changes sign under the space inversion operation, and so should the right-hand part. Therefore the tensor χ_{ilp} should change its sign when the coordinate inversion operation is applied. In a medium that has non-zero spin current tensor (such as magnetic medium with non-centrosymmetric magnetization distribution) it may be connected to the spin current

tensor that also changes its sign under the space inversion operation [37]:

$$\chi_{ilp} = \beta_{ijklp} J_{jk}, \quad (3)$$

where β_{ijklp} is the fifth rank tensor. Note that we use the spin current tensor here as a description of the non-centrosymmetric properties of the medium. The microscopic roots of this second harmonic generation effect may be not connected directly to this spin current. However, non-collinear magnetization gives rise to both effects in a similar way.

In an isotropic medium the form of β_{ijklp} is restricted to [40]:

$$\begin{aligned} \beta_{ijklp} = & A_1 e_{ijk} \delta_{lp} + A_2 e_{ijk} \delta_{ip} \\ & + A_3 e_{ijl} \delta_{kp} + A_4 e_{ikl} \delta_{jp}, \end{aligned} \quad (4)$$

assuming that the tensor should be symmetric with respect to the l and p indices (the corresponding symmetrizing terms are omitted). The e_{ijk} tensor in (4) is the completely antisymmetric Levi-Civita tensor, δ_{lp} is the Kronecker delta. Compiling the formula (4) with (1)–(3) we have for this case:

$$\begin{aligned} \mathbf{P}^{2\omega} = & B_1 \mathbf{P} (\mathbf{E}^\omega)^2 + B_2 \mathbf{E}^\omega (\mathbf{P} \cdot \mathbf{E}^\omega) \\ & + B_3 [\mathbf{E}^\omega \times [\mathbf{M} \times (\mathbf{E}^\omega \cdot \nabla) \mathbf{M}]], \end{aligned} \quad (5)$$

where B_l are the constants corresponding to A_l ($l = 1..3$), the term with A_4 is linearly dependent on that with $A_1..A_3$ and therefore is omitted, \mathbf{P} denotes the electric polarization that appears in a non-uniformly magnetized medium due to the flexo-magnetoelectric effect [32]:

$$\mathbf{P} = \alpha e_{ijk} \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_j} \right]_i \mathbf{e}_k, \quad (6)$$

α is a constant, \mathbf{e}_k stands for the k th unit vector in the Cartesian coordinate system.

An important symmetry point should be noted here, before proceeding to the calculations. It is known that the exchange interaction is invariant with respect to the rotations of the whole spin system [11, 40]. The spin current tensor J_{jk} is transformed as a vector with respect to the j index (in our definitions) under such rotations. The formula (1) satisfies the exchange interaction symmetry and hence may be caused by it. However, the equation (5) does not have such symmetry. Indeed, the rotation of the spin system leads to the rotation of the right-hand part of these equations while the left-hand part is not transformed. Thus, the second harmonic polarization that we are seeking for depends on the direction of the magnetization. Therefore it should be related to the spin-orbit interaction. We will further take this interaction into account by phenomenologically including the effects that have the spin-orbit origin such as the anomalous and spin Hall effect (see below).

3. Model and equations

In order to understand the origin of the effect described by (5) we consider the motion of the conduction electrons with the hydrodynamic equations [41] extended with the equations for spin [12, 42–44].

The Euler equation for the total electron velocity \mathbf{V} reads

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{e}{m} \mathbf{E}^\omega - \frac{e}{m} \mathbf{E} + \eta_H [\mathbf{V} \times \mathbf{M}] - \frac{e}{mc} [\mathbf{V} \times \mathbf{B}^\omega] - \frac{\mathbf{V}}{\tau_p}, \quad (7)$$

where \mathbf{B}^ω is the magnetic field of the wave, \mathbf{E} is the field induced by the wave, e is the absolute electron charge, m is its mass, η_H is the anomalous Hall constant (that has its roots in the spin-orbit interaction), τ_p is the momentum relaxation time, c is the velocity of light. The magnetic moment induced by the conduction electrons is neglected in (7). Besides, we do not take into account the spatial dispersion, and thus have $\mathbf{E}^\omega = \mathbf{E}_0 \exp(i\omega t)$, ω is the wave frequency (the Lorentz force is written in (7) with \mathbf{B}^ω representing an additional nonlinearity mechanism; however, this mechanism is not important in the scope of the current paper).

Equation (7) is accompanied by the standard law of conservation of mass:

$$\frac{\partial n}{\partial t} + \text{div} \mathbf{q} = 0, \quad (8)$$

where n is the electron density, and the electron flow \mathbf{q} is defined as:

$$\mathbf{q} = n\mathbf{V} = n\mathbf{v} + \alpha_{\text{SH}} e_{ijk} J_{ij} \mathbf{e}_k. \quad (9)$$

Here, a phenomenological term is added that describes the inverse spin Hall effect (which appears due to the spin-orbit interaction), α_{SH} is the inverse spin Hall effect constant. In (9), \mathbf{v} is a ‘normal’ electron velocity in the absence of the inverse spin hall effect (added for the convenience), e_{ijk} is the completely antisymmetric rank three tensor. It is important to note that we neglect the diffusion term in (9). One may check that the diffusion terms lead to higher-order derivatives of \mathbf{M} in an isotropic medium with no boundaries.

The induced electric field \mathbf{E} in (7) is determined by the Maxwell equation:

$$\text{div} \mathbf{E} = -4\pi e (n - n_0), \quad (10)$$

where n_0 is the equilibrium electron density (the wave field \mathbf{E}^ω is neglected since we do not take into account the spatial dispersion). The spin current tensor J_{ij} is determined as follows:

$$J_{ij} = s_i v_j + A \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_j} \right]_i, \quad (11)$$

where we phenomenologically add the equilibrium exchange spin current that is determined by the constant A ; s_i in (11) stands for the i th component of the electron spin density \mathbf{s} . We are seeking the effect linear with respect to the spin-orbit interaction. Hence there is no spin Hall effect in the equation for spin current (11); the diffusion term is also neglected in (11). In order to have the closed system of equations we add the spin conservation law [11]:

$$\frac{\partial \mathbf{s}}{\partial t} + \frac{\partial J_{ij}}{\partial x_j} \mathbf{e}_i + \frac{[\mathbf{s} \times \mathbf{M}]}{\tau_{\text{ex}}} + \frac{\mathbf{s} - \mathbf{s}_0}{\tau_s} = 0. \quad (12)$$

Here $\tau_{\text{ex}} = \frac{\hbar}{2J}$ is the exchange time that determines the spin precession period (J is the exchange constant), τ_s is the spin

relaxation time, and $\mathbf{s}_0 = \beta \mathbf{M}$ is the equilibrium average spin density of the electrons ($\beta \approx n_0 \frac{J}{\varepsilon_F}$ is a constant that characterizes the difference in the density of spin-up and spin-down electrons in the ferromagnet, ε_F is the Fermi energy).

We solve the system of equations (7)–(12) (see appendix). The second harmonic electric polarization is obviously connected to the double frequency electron flow $\mathbf{q}^{2\omega}$:

$$\mathbf{P}^{2\omega} = -\frac{e}{2i\omega} \mathbf{q}^{2\omega}. \quad (13)$$

The main approximations and restrictions of our model are summed in the list below.

- i The localized electrons induce the non-uniform magnetization $\mathbf{M}(\mathbf{r})$. We restrict ourselves to the first order derivatives of $\mathbf{M}(\mathbf{r})$, neglecting higher order derivatives.
- ii The delocalized conduction electrons are in charge of the optical response, including the non-linear effects.
- iii Only the linear in α_{SH} and η_H (linear in the spin-orbit interaction) terms are taken into account.

4. Calculations and discussion

We find the second harmonic electron flow that is proportional to the second order of \mathbf{M} and hence may be determined by the spin current. It has the following form:

$$\mathbf{q}^{2\omega} = \xi [\mathbf{E}^\omega \times [\mathbf{M} \times (\mathbf{E}^\omega \cdot \nabla) \mathbf{M}]], \quad (14)$$

where the constant ξ is linear within the spin-orbit interaction (proportional to α_{SH} , see below). Obviously, the relation (14) corresponds to the third term in the phenomenological expression (5) (determined by B_3). The terms determined by the static electric polarization \mathbf{P} do not exist in bulk in our hydrodynamic model. We suppose that such terms should appear if the boundaries are considered.

Calculations show that the constant ξ is the following (for more details, see appendix):

$$\xi = \frac{-\left(\frac{e}{m}\right)^2 \tau_{\text{ex}} \beta \alpha_{\text{SH}}}{\left(1 + \left(i\omega + \frac{1}{\tau_s}\right)^2 \tau_{\text{ex}}^2\right) \left(\left(\omega - \frac{\omega_p^2}{\omega}\right) - \frac{i}{\tau_p}\right)^2}, \quad (15)$$

here $\omega_p = \frac{4\pi n_0 e^2}{m}$ is the electron plasma frequency. As expected, it is proportional to the inverse spin Hall effect constant. The mechanism of its appearance follows from the calculations. The electrons oscillation in the electric field of the electromagnetic wave leads to a ‘non-adiabatic’ spin polarization that is proportional to $[\mathbf{M} \times (\mathbf{E}^\omega \cdot \nabla) \mathbf{M}]$. It appears in addition to the ‘adiabatic’ transfer of the spin by the moving (oscillating) electrons due to the precession about the local magnetization that is not parallel to this spin in the non-equilibrium state. Such ‘non-adiabatic’ spin is similar to that described by Aharonov and Stern for the electrons in equilibrium [45]. Together with the velocity that is proportional to \mathbf{E}^ω the ‘non-adiabatic’ spin polarization leads to the second harmonic spin current. The inverse spin Hall effect converts it to the double frequency electric current. The resonance at the

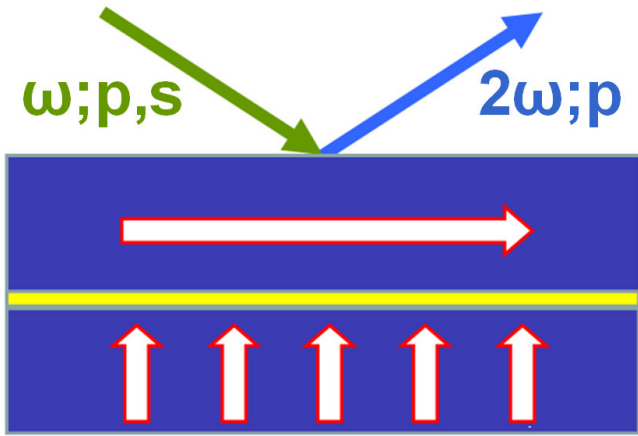


Figure 1. Possible scheme of the second harmonic generation effect in a non-collinear magnetic structure.

plasma frequency appears due to the oscillation of the charge density in the right side of the equation (10).

Note that although the inversion symmetry breaking is connected to the *equilibrium* spin current in the system [11] the mechanism described here appears from the *dynamic* spin current. As we have already mentioned, the inversion symmetry breaking in our system is caused by the non-uniform magnetization distribution which is non-centrosymmetric. This property of the magnetization may be described by a tensor (1) that has the properties of the spin current tensor: it changes sign under the space-inversion symmetry which is important for us. Therefore this tensor could be used to characterize the new effects that appear in such media and are related to the space-inversion symmetry breaking. For instance, we write the double frequency polarization in the form (5). This polarization does not exist in a centrosymmetric media for sure and therefore it contains the spin current as the term that describes the non-centrosymmetric properties. However the microscopic reasons of its appearance may be quite different. In our case, the dynamic spin current is responsible for it.

The considered effect may be observed in an artificial magnetic system that consists of two ferromagnetic layers with different types of anisotropy divided by a thin non-magnetic interlayer (figure 1). Such system has a non-collinear magnetization distribution in equilibrium. Besides, there may be the exchange interaction between two subsystems if the interlayer is thin enough [46]. The magnetization distribution may be controlled by applying an external magnetic field. The spin current here may be written as $J_{ij} = \tilde{u} [\mathbf{M}_1 \times \mathbf{M}_2]_j n_i^{12}$ where \mathbf{n}^{12} is a vector directed from the first layer to the second one (along the normal), $\mathbf{M}_{1,2}$ are the magnetizations of the layers, \tilde{u} is a constant. Therefore, we have an $[\mathbf{M}_1 \times \mathbf{M}_2]$ vector in addition to \mathbf{M}_1 and \mathbf{M}_2 . The polarization properties of the second harmonic generation from a surface for different directions of the magnetization vector are outlined in [47]. One may find that if the magnetizations of both layers lie in the pump plane of incidence there is no second harmonic signal in the p-polarization from the first or the second layer separately. However, the new effect that is determined by $[\mathbf{M}_1 \times \mathbf{M}_2]$ would give a p-polarized second harmonic wave for both s- and p-polarized

pump. Thus, the p-p or s-p combination of pump and second harmonic polarization is optimal for observation of the effect of second harmonic generation due to the spin currents in such system.

5. Conclusion

In summary, we theoretically investigate the second harmonic generation effect in a system with non-collinear magnetization distribution. The mechanism of the non-linear optical effect that is usually attributed to the spin current in such system is unraveled. It consists in the ‘non-adiabatic’ spin polarization of the conduction electrons which appears due to the oscillations of the electrons in the non-uniform magnetization forced by the electric field of the electromagnetic wave. This spin leads to the second harmonic spin current that is converted to the electric current via the inverse spin Hall effect. Thus, the equilibrium spin current is not the direct reason of the second harmonic generation here, although non-collinear magnetization gives rise to both effects. The analysis of the polarization properties of the second harmonic response shows that it could be observed in a multilayer non-collinear magnetic structure in a p-p or s-p combination of pump and second harmonic polarization.

Acknowledgments

The authors are thankful to T V Murzina for valuable discussions. This work was supported by the Russian Science Foundation (Grant No. 16-12-10340).

Appendix A

The solution of the equations in the extended hydrodynamic model

We solve the system of equations (7)–(12). The expansion of the variables that are involved into (7)–(12) as a power series of the electric field of the wave reads:

$$n = n^0 + n^\omega + n^{2\omega} + \dots, \quad (\text{A.1})$$

$$\mathbf{v} = \mathbf{v}^0 + \mathbf{v}^\omega + \mathbf{v}^{2\omega} + \dots, \quad (\text{A.2})$$

$$\mathbf{s} = \mathbf{s}^0 + \mathbf{s}^\omega + \mathbf{s}^{2\omega} + \dots, \quad (\text{A.3})$$

$$\mathbf{q} = \mathbf{q}^0 + \mathbf{q}^\omega + \mathbf{q}^{2\omega} + \dots, \quad (\text{A.4})$$

$$J_{ij} = J_{ij}^0 + J_{ij}^\omega + J_{ij}^{2\omega} + \dots \quad (\text{A.5})$$

Here we assume that the second-order terms oscillate at the double frequency, i.e. neglect the rectification terms that are beyond the scope of current paper. The terms linear with respect to the small parameters α_{SH} and η_H (which are supposed to be of the same order of value due to their spin-orbit roots) are found. We restrict ourselves to the second order with respect to the wave electric (or magnetic) field and to the first order with α_{SH} and η_H . Besides, we leave only \mathbf{M}

and its first-order spatial derivative, neglecting all higher-order derivatives and the powers of the first-order derivative. In these approximations, the solution (A.1)–(A.5) of the equations (7)–(12) restricted to the second order in \mathbf{E}^ω is the following.

$$n^0 = n_0, \mathbf{v}^0 = -\frac{\alpha_{\text{SH}} A}{n_0} e_{ijk} \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_j} \right]_i \mathbf{e}_k, \\ \mathbf{s}^0 = \mathbf{s}_0 = \beta \mathbf{M}, J_{ij}^0 = A \left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x_j} \right]_i, \mathbf{q}^0 = 0; \quad (\text{A.6})$$

$$\mathbf{v}^\omega = \frac{-\frac{e}{m}}{i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega}} \left(\mathbf{E}^\omega + \frac{\eta_H + \frac{\alpha_{\text{SH}} \beta \omega_p^2}{n_0}}{i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega}} [\mathbf{E}^\omega \times \mathbf{M}] \right), \\ \mathbf{s}^\omega = \frac{\beta \tau_{\text{ex}} \frac{e}{m}}{\left(i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega} \right) \left(1 + \tau_{\text{ex}}^2 \left(i\omega + \frac{1}{\tau_s} \right)^2 \right)} \\ \times \left(\tau_{\text{ex}} \left(i\omega + \frac{1}{\tau_s} \right) (\mathbf{E}^\omega \cdot \nabla) \mathbf{M} + [\mathbf{M} \times (\mathbf{E}^\omega \cdot \nabla) \mathbf{M}] \right), \quad (\text{A.7})$$

$$n^\omega = \frac{\frac{e}{m} (\mathbf{E}^\omega \cdot \text{curl} \mathbf{M})}{i\omega \left(\frac{1}{\tau_p} + i\omega \right)} \frac{\alpha_{\text{SH}} \beta \left(i\omega + \frac{1}{\tau_p} \right) - n_0 \eta_H}{i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega}}, \\ J_{ij}^\omega = (\mathbf{s}_0)_i (\mathbf{v}^\omega)_j, \mathbf{q}^\omega = n_0 \mathbf{v}^\omega + \alpha_{\text{SH}} [\mathbf{s}_0 \times \mathbf{v}^\omega]; \\ \mathbf{v}^{2\omega} = Q_1 [\mathbf{E}^\omega \times ((\mathbf{E}^\omega \cdot \nabla) \mathbf{M})] + Q_2 [\mathbf{E}^\omega \times \mathbf{B}^\omega] \\ + Q_3 [[\mathbf{E}^\omega \times \mathbf{B}^\omega] \times \mathbf{M}],$$

$$Q_1 = \frac{-\left(\frac{e}{m}\right)^2 \left(\eta_H + \frac{\alpha_{\text{SH}} \beta}{n_0} \left(\frac{\omega_p^2}{i\omega} - i\omega - \frac{1}{\tau_p} \right) \right)}{\left(i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega} \right)^3 \left(2i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{2i\omega} \right)}, \\ Q_2 = \frac{\left(\frac{e}{m}\right)^2}{c \left(i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega} \right) \left(2i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{2i\omega} \right)}, \\ Q_3 = \frac{-\left(\frac{e}{m}\right)^2 \left(\eta_H + \alpha_{\text{SH}} \beta \frac{\omega_p^2}{i\omega n_0} \right)}{c \left(i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{i\omega} \right)^2 \left(2i\omega + \frac{1}{\tau_p} + \frac{\omega_p^2}{2i\omega} \right)} \frac{3i\omega + \frac{2}{\tau_p}}{2i\omega + \frac{1}{\tau_p}}, \quad (\text{A.8})$$

$$\mathbf{s}^{2\omega} = 0, n^{2\omega} = -\frac{1}{2i\omega} \text{div} (n_0 \mathbf{v}^{2\omega} + \alpha_{\text{SH}} \beta [\mathbf{M} \times \mathbf{v}^{2\omega}]), \\ J_{ij}^{2\omega} = (\mathbf{s}_0)_i (\mathbf{v}^{2\omega})_j + (\mathbf{s}^\omega)_i (\mathbf{v}^\omega)_j, \\ \mathbf{q}^{2\omega} = n_0 \mathbf{v}^{2\omega} + n^\omega \mathbf{v}^\omega + \alpha_{\text{SH}} ([\mathbf{s}_0 \times \mathbf{v}^{2\omega}] + [\mathbf{s}^\omega \times \mathbf{v}^\omega]).$$

Although we find the solution in bulk, we should take into account the boundaries of the sample in order to obtain these results. In this case, the static polarization \mathbf{P} which describes the flexo-magnetoelectric effect and the electric field connected to it via the Maxwell equations compensates the flow that appears due to the inverse spin Hall effect. The ‘normal’ part of electron velocity \mathbf{v}^0 is therefore nonzero. However, full electron velocity and the electron flow is zero.

It is important to note that the first term in \mathbf{s}^ω linear in \mathbf{M} corresponds to the adiabatic spin transfer under the application of the oscillating electric field to the non-uniformly magnetized medium. The second term in \mathbf{s}^ω that is quadratic with

respect to \mathbf{M} appears due to the non-adiabatic processes in the medium.

The expression for $\mathbf{q}^{2\omega}$ contains the terms linear in \mathbf{M} . First, those are the terms that do not contain the differentiation operator and hence may appear in a uniformly magnetized medium. These terms arise from the Lorentz force due to the magnetic field of the wave in an infinite system. Next, there are terms that contain the derivative of \mathbf{M} and appear in a non-uniformly magnetized media. These terms are usually attributed to the toroidal moment of the system [48–50]. Besides, the fourth term in its right-hand part ($\alpha_{\text{SH}} [\mathbf{s}^\omega \times \mathbf{v}^\omega]$) gives the second-order in \mathbf{M} term that is connected to the spin current in the system. The w_1 coefficient in (14) determining it may be easily obtained from the expression for $\mathbf{q}^{2\omega}$ in (A.8). It appears due to the non-adiabatic processes mentioned above.

ORCID iDs

E A Karashtin  <https://orcid.org/0000-0003-3888-3112>

References

- [1] Žutić I, Fabian J and Das Sarma S 2004 *Rev. Mod. Phys.* **76** 323
- [2] Stamps R L *et al* 2014 *J. Phys. D: Appl. Phys.* **47** 333001
- [3] Katine J A and Fullerton E E 2008 *J. Magn. Magn. Mater.* **320** 1217
- [4] Parkin S S P, Hayashi M and Thomas L 2008 *Science* **320** 190
- [5] Atkinson D, Eastwood D S and Bogart L K 2008 *Appl. Phys. Lett.* **92** 022510
- [6] Sato K and Saitoh E 2016 *Spintronics for Next Generation Innovative Devices* (Chichester: Wiley) pp 1–255
- [7] Johnson M and Silsbee R H 1985 *Phys. Rev. Lett.* **55** 1790
- [8] Shikin A M, Rybkina A A, Rybkin A G, Klimovskikh I I, Skirdkov P N, Zvezdin K A and Zvezdin A K 2014 *Appl. Phys. Lett.* **105** 042407
- [9] Tserkovnyak Y, Brataas A and Bauer G E W 2002 *Phys. Rev. Lett.* **88** 117601
- [10] Brataas A, Tserkovnyak Y, Bauer G E W and Halperin B I 2002 *Phys. Rev. B* **66** 060404
- [11] Sonin E 2010 *Adv. Phys.* **59** 181
- [12] Sinova J, Valenzuela S O, Wunderlich J, Back C H and Jungwirth T 2015 *Rev. Mod. Phys.* **87** 1213
- [13] Saitoh E, Ueda M, Miyajima H and Tataru G 2006 *Appl. Phys. Lett.* **88** 182509
- [14] Valenzuela S O and Tinkham M 2006 *Nature* **442** 176
- [15] Zhao H, Loren E J, van Driel H M and Smirl A L 2006 *Phys. Rev. Lett.* **96** 246601
- [16] Li J *et al* 2016 *Phys. Rev. Lett.* **117** 076602
- [17] Liang F, Wang J, Yang Y H and Chan K S 2008 *J. Appl. Phys.* **104** 113701
- [18] Lou X, Adelman C, Crooker S A, Garlid E S, Zhang J, Madhukar Reddy K S, Flexner S D, Palmström C J and Crowell P A 2007 *Nat. Phys.* **3** 197
- [19] Tombros N, Jozsa C, Popinciuc M, Jonkman H T and van Wees B J 2007 *Nature* **448** 571
- [20] van't Erve O M J, Hanbicki A T, Holub M, Li C H, Awo-Affouda C, Thompson P E and Jonker B T 2007 *Appl. Phys. Lett.* **91** 212109
- [21] Wang J and Chan K S 2006 *Phys. Rev. B* **74** 035342
- [22] König J, Bønsager M C and MacDonald A H 2001 *Phys. Rev. Lett.* **87** 187202

- [23] Schütz F, Kollar M and Kopietz P 2003 *Phys. Rev. Lett.* **91** 017205
- [24] Heurich J, König J and MacDonald A H 2003 *Phys. Rev. B* **68** 064406
- [25] Splittstoesser J, Governale M and Zülicke U 2003 *Phys. Rev. B* **68** 165341
- [26] Wang B, Peng J, Xing D Y and Wang J 2005 *Phys. Rev. Lett.* **95** 086608
- [27] Chshiev M, Manchon A, Kalitsov A, Ryzhanova N, Vedyayev A, Strelkov N, Butler W H and Diény B 2015 *Phys. Rev. B* **92** 104422
- [28] Chen W, Horsch P and Manske D 2014 *Phys. Rev. B* **89** 064427
- [29] Shen Y G and Yang Z H 2007 *Europhys. Lett.* **78** 17003
- [30] Wang J and Chan K S 2007 *J. Phys.: Condens. Matter* **19** 236215
- [31] Bar'yakhtar V G, L'vov V A and Yablonskii D A 1983 *Pis'ma Zh. Eksp. Teor. Fiz.* **37** 565
- [32] Katsura H, Nagaosa N and Balatsky A V 2005 *Phys. Rev. Lett.* **95** 057205
- [33] Bruno P and Dugaev V K 2005 *Phys. Rev. B* **72** 241302
- [34] Mills D L and Dzyaloshinskii I E 2008 *Phys. Rev. B* **78** 184422
- [35] Logginov A S, Meshkov G A, Nikolaev A V, Nikolaeva E P, Pyatakov A P and Zvezdin A K 2008 *Appl. Phys. Lett.* **93** 182510
- [36] Veshchunov I S, Mironov S V, Magrini W, Stolyarov V S, Rossolenko A N, Skidanov V A, Trebbia J B, Buzdin A I, Tamarat P and Lounis B 2015 *Phys. Rev. Lett.* **115** 027601
- [37] Wang J, Zhu B F and Liu R B 2010 *Phys. Rev. Lett.* **104** 256601
- [38] Werake L K and Zhao H 2010 *Nat. Phys.* **6** 875
- [39] Landau L D and Lifshitz E M 1980 *Course of Theoretical Physics (Statistical Physics, Part 2: Theory of the Condensed State vol 9)* (Oxford: Butterworth-Heinemann)
- [40] Landau L D and Lifshitz E M 1984 *Course of Theoretical Physics (Electrodynamics of Continuous Media vol 8)* (Oxford: Butterworth-Heinemann)
- [41] Boyd R W 2008 *Nonlinear Optics* 3rd edn (New York: Academic)
- [42] Dyakonov M I and Perel V I 1971 *Phys. Lett. A* **35** 459
- [43] Dyakonov M I and Perel V I 1971 *JETP Lett.* **13** 467
- [44] Shpiro A, Levy P M and Zhang S 2003 *Phys. Rev. B* **67** 104430
- [45] Aharonov Y and Stern A 1992 *Phys. Rev. Lett.* **69** 3593
- [46] Demidov E S, Gusev N S, Budarin L I, Karashtin E A, Mironov V L and Fraerman A A 2016 *J. Appl. Phys.* **120** 173901
- [47] Rzhevsky A A, Krichevtsov B B, Bürgler D E and Schneider C M 2007 *Phys. Rev. B* **75** 144416
- [48] Petukhov A V, Lyubchanskii I L and Rasing T 1997 *Phys. Rev. B* **56** 2680
- [49] Krutyanskiy V L, Kolmychek I A, Gribkov B A, Karashtin E A, Skorohodov E V and Murzina T V 2013 *Phys. Rev. B* **88** 094424
- [50] Kolmychek I A, Krutyanskiy V L, Murzina T V, Sapozhnikov M V, Karashtin E A, Rogov V V and Fraerman A A 2015 *J. Opt. Soc. Am. B* **32** 331