Drastic Suppression of Noise-Induced Errors in Underdamped Long Josephson Junctions

Andrey L. Pankratov,[1,2,*] Anna V. Gordeeva,[2,1] and Leonid S. Kuzmin[2,3]

[1]Institute for Physics of Microstructures of RAS, Nizhny Novgorod 603950, Russia
[2]Laboratory of Cryogenic Nanoelectronics, Nizhny Novgorod State Technical University, Nizhny Novgorod 603950, Russia
[3]Chalmers University of Technology, SE-41296 Gothenburg, Sweden

(Received 22 April 2012; published 24 August 2012)

The fluctuational propagation of solitons (magnetic fluxons) in long Josephson junctions is studied both numerically and analytically. It is demonstrated that operation in conditions where solitons are subjected to Lorentz contraction for a significant part of the junctions length leads to drastic suppression of thermal jitter at the output junction end. Specifically, for large-to-critical damping and small values of bias current, the physically obvious dependence of the jitter versus length $\sigma \sim \sqrt{L}$ is confirmed, while for small damping starting from the experimentally relevant $\alpha = 0.1$ and below, strong deviation from $\sigma \sim \sqrt{L}$ is observed, up to nearly complete independence of the jitter versus length, which is supported by the obtained theory.

DOI: 10.1103/PhysRevLett.109.087003 PACS numbers: 74.40.−n, 05.40.−a, 05.45.Yv

The solitons, contracting during their propagation, occur naturally in a wide variety of solids and fluids, from macroscopic to microscopic scales, such as, e.g., ultrashort light pulses in nonlinear active media [1,2], solitons in a shallow water [3], magnetic fluxons in long Josephson junctions [4]. During transfer, the solitons are perturbed by various fluctuations, affecting their dynamics. Fluctuations restrict useful properties of electronic devices, such as the sensitivity of detectors and receivers. However, to our knowledge, the effect of the nonstationary soliton dynamics on fluctuations in systems has not been studied, in particular due to technical complexity to perform the corresponding measurements: the study of the jitter of the light pulses is restricted by the sensitivity of the existing equipment, while it is difficult to collect proper statistics studying the dynamics of nonlinear waves in shallow water. To this respect, the Josephson junctions serve as an ideal object to study, because there are techniques to create identical magnetic fluxons and the existing equipment allows us to measure the jitter in Josephson electronic devices [5]. Besides, in recent years superconducting circuits have attracted considerable interest as promising devices for quantum computations [6–9]. The advantage of superconducting circuits in comparison with other types of qubits is the possibility of combining both the qubits and the readout electronics in one chip, which is very promising, because it allows the elimination of expensive external readout equipment as well as minimization of parasitic capacitances and inductances, affecting the readout pulses. Recently [10] a suggestion has been made to use for the readout the well-elaborated rapid single flux quantum (RSFQ) devices, redesigned in a special way. Here the main idea is the ballistic propagation of magnetic fluxons along two separated Josephson transmission lines (JTLs) [10–13], consisting of either one long or a series of short junctions having weak damping (see Fig. 1). If one JTL is inductively coupled to the qubit, the magnetic field of the qubit will change the speed of the fluxon (soliton), traveling from one end of the JTL to the other, and will thus change the fluxon transmission time. This difference of fluxon transmission times in two JTLs can be effectively measured using the existing RSFQ circuitry [single flux quantum (SFQ) receiver in Fig. 1], while the proper fluxons can be produced in the SFQ driver.

It is obvious that by increasing the JTL length, the precision of the time difference measurement can be improved. However, different types of fluctuations restrict the readout precision because they lead to the jitter of traveling fluxons. The importance of studying jitter in Josephson junctions was first understood in Ref. [14]. Later, jitter in short junctions has been studied both analytically and numerically [15–18]. However, as has been understood and experimentally verified [5], for a series of junctions the jitter increases as $\sigma \sim \sqrt{N}$, where N is the number of junctions. Therefore, for JTLs the jitter must increase as $\sigma \sim \sqrt{L}$ [12] (where L is the JTL length), which is assumed to be a fundamental limitation due to broadband fluctuations. However, to our knowledge, except for the theory of Ref. [12], the jitter in underdamped JTLs and long Josephson junctions has not been studied either theoretically or experimentally. The experimental results devoted to RSFQ [5] cannot be directly applied as they are obtained for critically damped junctions rather than for

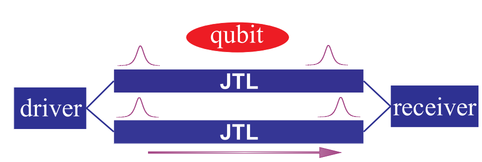

FIG. 1 (color online). Illustration of the ballistic readout from the qubit using two Josephson transmission lines, where the top one is inductively coupled to the qubit.

underdamped junctions. This Letter, therefore, is devoted to a detailed theoretical study of the jitter versus damping and the length in an underdamped long Josephson junction. Here we demonstrate that the fundamental limitation $\sigma \sim \sqrt{L}$ can be overcome at certain conditions. In this Letter, we restrict ourselves by a classical model of fluxons, in which thermal fluctuations of the current noise are dominant and neglected by various quantum effects of soliton dynamics [19,20].

Let us consider the long Josephson tunnel junction (JTJ) in the frame of the sine—Gordon equation

$$\phi_{tt} + \alpha\phi_t - \phi_{xx} = i - \sin(\phi) + i_f(x, t), \qquad (1)$$

where indices t and x denote temporal and spatial derivatives. Space and time are normalized to the Josephson penetration length λ_J and to the inverse plasma frequency ω_p^{-1}, respectively, $\alpha = \omega_p/\omega_c$ is the damping, $\omega_p = \sqrt{2eI_c/\hbar C}$, $\omega_c = 2eI_cR_N/\hbar$, I_c is the critical current, C is the JTJ capacitance, R_N is the normal state resistance, i is the dc overlap bias current density, normalized to the critical current density J_c, and $i_f(x, t)$ is the fluctuational current density. If the critical current density is fixed and the fluctuations are treated as white Gaussian noise with zero mean, its correlation function is $\langle i_f(x, t) i_f(x', t')\rangle = 2\alpha\gamma\delta(x - x')\delta(t - t')$, where $\gamma = I_T/(J_c\lambda_J)$ is the dimensionless noise intensity [21], $I_T = 2ekT/\hbar$ is the thermal current, e is the electron charge, $\hbar$ is the Planck constant, k is the Boltzmann constant, and T is the temperature.

The boundary conditions that describe coupling to the environment have the form [22]:

$$\phi(0, t)_x + r_L c_L \phi(0, t)_{xt} - c_L \phi(0, t)_{tt} = \Gamma, \qquad (2)$$

$$\phi(L, t)_x + r_R c_R \phi(L, t)_{xt} + c_R \phi(L, t)_{tt} = \Gamma. \qquad (3)$$

Here $\Gamma = 0$ is the normalized magnetic field, and L is the dimensionless length of JTJ. The terms with the dimensionless capacitances and resistances, $c_{L,R}$ and $r_{L,R}$, are the RC load of a JTJ placed at the left (input) and at the right (output) ends, respectively.

For simplicity, we assume that the bias current density is uniformly distributed along the space $i(x) = \mathrm{const}$. As initial condition, a kink $\phi(x, t) = 4\arctan[\exp(x - x_0)]$ inside the junction is taken (for $x_0 = 5$). Temporal and spatial intervals are chosen $\Delta t = \Delta x = 0.01$, and it has been verified that further decrease in the steps does not change the results. Two values of RC load were tested: a mismatched case where simple boundary conditions $d\phi(x = 0)/dx = d\phi(x = L)/dx = 0$ were used, and a perfectly matched case $r_L = r_R = 1$, $c_L = c_R = 100$. Because the results are nearly the same, the curves for the perfectly matched case (to suppress reflected waves from the junction ends, as required in experiments) are shown.

Two definitions of the fluxon traveling time τ and the jitter σ are used. One is the usual mean first passage time of the boundary, where the random time at which the soliton hits the right junction end is computed and then the mean value and the standard deviation are calculated by averaging over 1000 realizations. Another definition is based on the notion of the integral relaxation time [21,23]: if the probability of finding the soliton inside the junction is computed $P(t) = \int_0^L W(x, t)dx$, the mean traveling time and the standard deviation (jitter) are

$$\tau = \langle t\rangle = \int_0^\infty t w(t)dt, \qquad \sigma = \sqrt{\langle t^2\rangle - \langle t\rangle^2},$$
$$w(t) = \frac{\partial P(t)}{\partial t[P(\infty) - P(0)]}. \qquad (4)$$

In the limit of small noise $\gamma \leq 0.1$ both definitions give the same results within the calculation precision [23].

In Fig. 2 the plots of the jitter versus JTL length are shown for various values of damping α and bias current i, and for noise intensity $\gamma = 0.001$.

While the two upper curves are well fitted by $\sqrt{L}$ dependence, one can see that by variation of damping α and bias current i it is possible to suppress the growing of jitter with length to $\sqrt[4]{L}$ law and for some parameters (e.g., $\alpha = 0.01$ and $i = 0.3$) jitter is even nearly independent of L, which is the main and quite surprising result of this Letter. The tendency is such that for a fixed α one needs to increase i to weaken the $\sigma(L)$ dependence or if the bias current is fixed the same effect appears with decrease of α.

In order to understand the origin of the observed result, it is helpful to consider the evolution of the soliton shape $H(x) = d\phi(x)/dx$, plotted at different instants of time. In Fig. 3, one can see the snapshots of the fluxon during its motion along the JTL. Analyzing its dynamics for different values of damping and bias current it has been noted that when the soliton shape weakly changes with time $\sigma \sim \sqrt{L}$ dependence is observed. But if the damping is small (see,

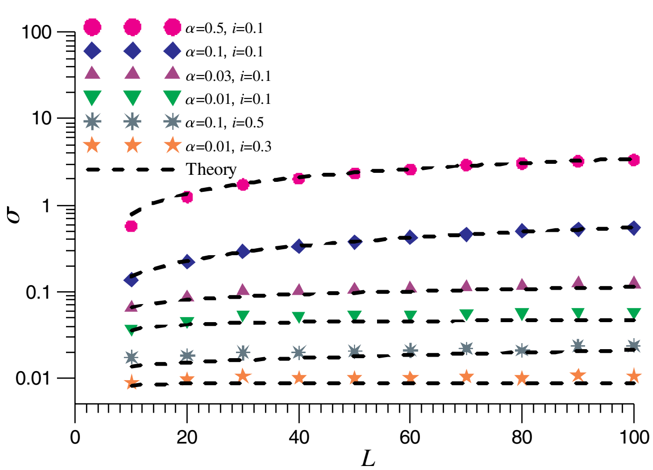

FIG. 2 (color online). Jitter versus JTL length for various values of damping α and bias current i for $\gamma = 0.001$. Symbols: simulations; dashed curves: theory [Eqs. (4) and (10)].

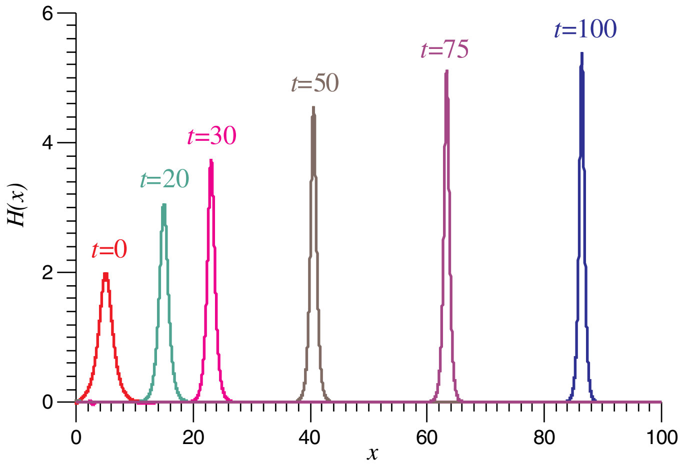

FIG. 3 (color online). Snapshots of the soliton, accelerating and contracting during the motion along JTL, $i = 0.1$ and $\alpha = 0.03$.

e.g., Fig. 3 for $i = 0.1$ and $\alpha = 0.03$), soliton acceleration and its Lorentz contraction lead to weaker dependence of jitter on the junction length. Considering this as the main explanation for violation of $\sigma \sim \sqrt{L}$ dependence of the jitter, we performed analytical analysis which allowed us to obtain a general expression for σ, describing its dependence on length in a broad range of junction parameters. The only theory known so far [12] does not take into account the nonstationarity of the fluxon dynamics and therefore does not describe the observed effect.

Starting from Eq. (1), one can write the equation for the momentum $p(t) = -\frac{1}{8}\int_{-\infty}^{+\infty}\phi_x\phi_t dx$ of the center of the soliton [4] taking account of the effect of noise [24]

$$\frac{dp}{dt} = -\alpha p + \frac{1}{4}\pi i + \xi(t), \tag{5}$$

where $p = v/\sqrt{1-v^2}$, $v(t)$ is the fluxon velocity, and the noise intensity $\langle\xi(t)\xi(t')\rangle = (\alpha\gamma/\sqrt[4]{1-v^2})\delta(t-t')$ with γ the same as in Eq. (1). Directly from Eq. (5), neglecting the effect of noise, one can derive the expression for the fluxon velocity $v(t)$ for $v(t=0) = 0$

$$v(t) = \frac{1}{\sqrt{1 + \frac{1}{[\frac{\pi i}{4\alpha}(1-e^{-\alpha t})]^2}}}. \tag{6}$$

From Eq. (5) one can derive the following equation for the location of the center of the fluxon $X(t)$

$$\ddot{X}(t) = -\alpha\dot{X}(t)(1-v^2) + \left[\frac{1}{4}\pi i + \xi(t)\right]\sqrt{(1-v^2)^3}, \tag{7}$$

where $\ddot{X}(t)$ and $\dot{X}(t)$ stand for temporal derivatives. This equation is too complex to be solved analytically; therefore, we will use the following approximations. First, let us neglect the random component of velocity (which can be done in the limit of small noise), so $v(t)$ is described by Eq. (6). Second, let us also neglect the term $(1-v^2)$ (which leads to effective decrease of damping) in front of the first temporal derivative $\dot{X}(t)$. Then Eq. (7) transforms into the equation of massive Brownian particles, but with the noise intensity, depending on time. This Brownian diffusion is described by the Gaussian probability density, since the noise $\xi(t)$ is Gaussian. Therefore, one can use the approach described in chapters 1 and 3 of Ref. [25], and can get the following expression for the variance $D(t)$ of the process $X(t)$, taking into account the nonstationarity of the noise intensity

$$D(t) = \frac{\gamma}{4\alpha}\int_0^t [1 - 2e^{-\alpha y} + e^{-2\alpha y}][1 - v^2(y)]^{5/2} dy, \tag{8}$$

where $v(y)$ is given by Eq. (6). Analogically, the mean $m(t)$ of $X(t)$ can be derived [with $X(t=0) = X_0$]

$$m(t) = X_0 + \int_0^t v(y)dy. \tag{9}$$

By substituting the mean and the variance into the Gaussian probability distribution one can obtain the probability of finding the soliton inside the junction

$$P(t) = 1 - \mathrm{erfc}\left[(L - m(t))/\sqrt{2D(t)}\right]/2. \tag{10}$$

By substituting $P(t)$ into Eq. (4), the mean traveling time τ and the standard deviation σ can be computed.

The comparison of the theory and simulations is presented in Figs. 2 and 4. The theory confirms violation of $\sqrt{L}$ dependence of the jitter both qualitatively and quantitatively. One can see, however, that for smaller damping the theory underestimates the numerical results (by 10% in the worst case) due to simplifications of Eq. (7).

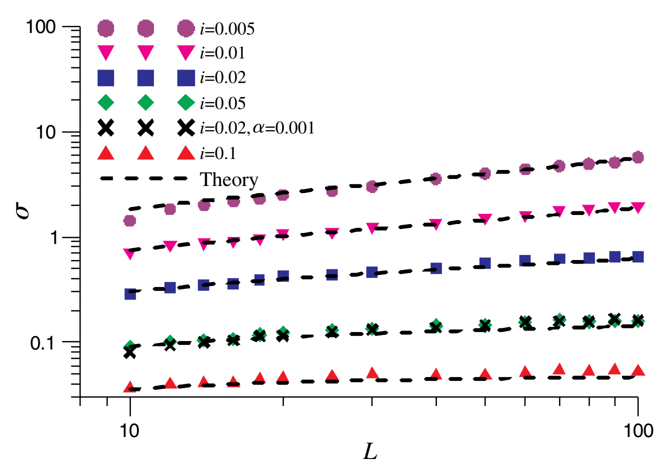

FIG. 4 (color online). Jitter versus JTL length for different values of bias current i and $\gamma = 0.001$, $\alpha = 0.01$ for all curves except the curve with crosses ($\alpha = 0.001$, $i = 0.02$). Symbols: simulations; dashed curves: theory [Eqs. (4) and (10)]. The dependence changes from $\sqrt{L}$, (circles) to $\sqrt[4]{L}$ (diamonds and crosses).

Let us perform calculations for fixed small value of damping $\alpha = 0.01$ and for various bias current values (see Fig. 4). It is seen that for small values of bias current the dependence $\sigma \sim \sqrt{L}$ is maintained, while for larger values starting from $i = 0.01$ the deviation is observed. It should be noted, however, that one must think about the thermal budget of both qubits and the readout electronics, trying to decrease the total heat, because usually cryostats have rather low thermal power at low temperatures. Nevertheless, the bias current values of order $i = 0.01$, at which the deviation from the dependence $\sigma \sim \sqrt{L}$ is observed, seem to be rather low to maintain low heat emission. It is known that with decreasing the temperature of the JTL from 4 K to 50 mK the damping decreases by one order of magnitude [13]. This allows operating with even smaller bias currents (see Fig. 4, curve with crosses) keeping the same order of jitter.

The main quantity of interest is the noise-to-signal ratio, which is the ratio between the jitter σ and the difference $\Delta\tau$ between the traveling times of solitons in two JTLs, see Fig. 1. While the calculation of $\Delta\tau$ is out of the scope of this Letter, we can estimate it as $\Delta\tau \approx 0.1\tau$, where τ is the traveling time in the reference line, not connected to a qubit. In fact, $\Delta\tau$ must be smaller than 0.1τ to minimize the back action of the detector on a qubit, but let us take this value as a rough estimate. Computing the ratio $\sigma/\Delta\tau$, we should note that the dependence of the curve is changing from $1/\sqrt{L}$ for $\alpha = 0.001$, $i = 0.002$ to $1/L$ for $\alpha = 0.01$, $i = 0.3$. It is important to mention that for $L = 100$ and $\alpha = 0.001$, $i = 0.002$, the readout error $\sigma/\Delta\tau$ becomes comparable with 0.1, which for a Gaussian variable is on the border of an acceptable level [26], while for $\alpha = 0.01$, $i = 0.3$ the error decreases by two orders of magnitude, allowing us to resolve even the $\Delta\tau \approx 0.001\tau$ time difference. Obviously, the readout error will never vanish due to jitter of the SFQ driver and receiver, as well as due to jitter coming from the technological inhomogeneity of the JTL. Nevertheless, the obtained results are exciting from a fundamental point of view, but also intriguing from the point of view of possible applications, because they allow the readout error suppression by increasing the JTL length.

In conclusion, the thermal jitter of transmission of magnetic fluxons in long Josephson junctions is studied. While for large-to-critical damping and small values of bias current the known dependence of the jitter versus length $\sigma \sim \sqrt{L}$ is confirmed, for smaller damping the strong deviation from $\sigma \sim \sqrt{L}$ is observed, up to nearly complete independence of the jitter versus length, which is explained by Lorentz contraction of solitons and supported by the obtained theory.

The work is supported by RFBR project 09-02-00491, Dynasty Foundation, Human Capital Foundation, and by the Act 220 of the Russian Government (project 25).

*alp@ipm.sci-nnov.ru

[1] R. A. Bartels, T. C. Weinacht, N. Wagner, M. Baertschy, C. H. Greene, M. M. Murnane, and H. C. Kapteyn, Phys. Rev. Lett. **88**, 013903 (2001).
[2] M. A. Martyanov, G. A. Luchinin, A. K. Potemkin, and E. A. Khazanov, Quantum Electron. **38**, 103 (2008).
[3] A. Slunyaev, C. Kharif, E. Pelinovsky, and T. Talipova, Physica (Amsterdam) **173D**, 77 (2002).
[4] D. W. McLaughlin and A. C. Scott, Phys. Rev. A **18**, 1652 (1978).
[5] H. Terai, Z. Wang, Y. Hishimoto, S. Yorozu, A. Fujimaki, and N. Yoshikawa, Appl. Phys. Lett. **84**, 2133 (2004); H. Terai, Y. Hashimoto, S. Yorozu, A. Fujimaki, N. Yoshikawa, and Z. Wang, IEEE Trans. Appl. Supercond. **15**, 364 (2005).
[6] Y. Makhlin, G. Schon, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[7] A. N. Korotkov, Quantum Inf. Process. **8**, 51 (2009).
[8] J. M. Martinis, Quantum Inf. Process. **8**, 81 (2009).
[9] Y. A. Pashkin, O. Astafiev, T. Yamamoto, Y. Nakamura, and J. S. Tsai, Quantum Inf. Process. **8**, 55 (2009).
[10] V. K. Semenov and D. V. Averin, IEEE Trans. Appl. Supercond. **13**, 960 (2003).
[11] D. V. Averin, K. Rabenstein, and V. K. Semenov, Phys. Rev. B **73**, 094504 (2006).
[12] A. Fedorov, A. Shnirman, G. Schon, and A. Kidiyarova-Shevchenko, Phys. Rev. B **75**, 224504 (2007).
[13] A. Herr, A. Fedorov, A. Shnirman, E. Ilichev, and G. Schon, Supercond. Sci. Technol. **20**, S450 (2007).
[14] A. V. Rylyakov and K. K. Likharev, IEEE Trans. Appl. Supercond. **9**, 3539 (1999).
[15] A. L. Pankratov and B. Spagnolo, Phys. Rev. Lett. **93**, 177001 (2004).
[16] V. K. Semenov and A. Inamdar, IEEE Trans. Appl. Supercond. **15**, 435 (2005).
[17] A. V. Gordeeva and A. L. Pankratov, Appl. Phys. Lett. **88**, 022505 (2006).
[18] A. V. Gordeeva and A. L. Pankratov, J. Appl. Phys. **103**, 103913 (2008).
[19] A. Shnirman, E. Ben-Jacob, and B. A. Malomed, Phys. Rev. B **56**, 14677 (1997); A. Wallraff, A. Lukashenko, J. Lisenfeld, A. Kemp, M. V. Fistul, Y. Koval, and A. V. Ustinov, Nature (London) **425**, 155 (2003).
[20] Z. Hermon, A. Shnirman, and E. Ben-Jacob, Phys. Rev. Lett. **74**, 4915 (1995); A. Shnirman, Z. Hermon, L. Vaidman, and E. Ben-Jacob, Phys. Rev. A **52**, 3541 (1995).
[21] K. G. Fedorov and A. L. Pankratov, Phys. Rev. B **76**, 024504 (2007).
[22] A. L. Pankratov, A. S. Sobolev, V. P. Koshelets, and J. Mygind, Phys. Rev. B **75**, 184516 (2007).
[23] A. N. Malakhov and A. L. Pankratov, Adv. Chem. Phys. **121**, 357 (2002).
[24] E. Joergensen, V. P. Koshelets, R. Monaco, J. Mygind, M. R. Samuelsen, and M. Salerno, Phys. Rev. Lett. **49**, 1093 (1982).
[25] W. T. Coffey, Yu. P. Kalmykov, and J. T. Waldron, *The Langevin Equation* (World Scientific, Singapore, 1996).
[26] A. A. Smirnov and A. L. Pankratov, Phys. Rev. B **82**, 132405 (2010).